

MATHLINKS: ESSENTIALS

PROPORTIONAL REASONING 3 STUDENT PACKET

These files are intended to support English learners. They include unformatted text from the packet so that translation (by an expert or by a computer program) will be more convenient. They are provided “as is” at no cost by the Center for Mathematics and Teaching.

MY WORD BANK (page 0)

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. (See section 3.5.) Key mathematical vocabulary is underlined throughout the packet.

cross-multiplication property / proportion / proportional relationship / scale / scale drawing
/ scale factor /

TWINKIE, THE DOG (page 1)

Twinkie, the Jack Russell Terrier, pops balloons. Will she break the world record? Follow your teacher’s directions to learn more about this amazing dog.

PROPORTIONAL REASONING (page 2)

We will use proportional reasoning strategies to solve problems.

Problems 1-4: Compute.

Problems 5-7: Use your knowledge of equivalent fractions to solve for x :

Problems 8-9: Find the cost. Try using sense-making strategies. See section 3.5 for ideas as needed.

8. If one pencil costs 35¢, what is the cost of 4 pencils?

9. If 6 pencils cost \$2.40, what is the cost of 1 pencil?

ART SUPPLIES (page 3)

Mrs. Carter is buying art supplies. Help her determine the cost and quantities for some items she needs. Assume costs and quantities are in a proportional relationship. Find this phrase in section 3.5 and record its definition in the Word bank. Then follow your teacher’s directions.

PRACTICE 1 (page 4)

Solve these problems using the given strategy. Check your work using another strategy of your choice.

1. Use a table. Jake's car used 15 gallons of gas to travel 330 miles
 - a. If the car continues to consume gas at this same rate, how far can the car go on 20 gallons?
 - b. How many miles per gallon does the car get?
2. Use a double number line. Angie paid \$14 for 4 gallons of gas
 - a. At this rate, how many gallons of gas can she buy for \$35?
 - b. How much will 14 gallons of gas cost?
 - c. What is the price per gallon?

PRACTICE 2 (page 5)

Solve these problems using the strategy given. Check your work using another strategy of your choice.

1. Use a table or a double number line.
Samara biked 6 miles in 30 minutes.
 - a. At that rate, how far could she go in 2 hours?
 - b. At that rate, how far could she go in 1 hour?
 - c. At that rate, how long would it take her to go 15 miles?
2. Use some form of arithmetic, such as a unit rate or a chunking strategy.
Greg is training for a marathon. He ran 21 miles in $3\frac{1}{2}$ hours
 - a. At that pace, how far did he run in one hour?
 - b. At that pace, *about* how long will it take him to run the marathon (26.2 miles)?

PROPERTIES OF PROPORTIONS (page 6)

Follow your teacher's directions to learn about proportions. Find this word in section 3.5 and record its definition in the Word bank. Four baseballs cost \$_____.

PRACTICE 3 (page 7)

Problems 1-6: Use equivalent fractions or the cross-multiplication property to solve each equation.

7. Some students explored the equation $3/5=6/10$, and rewrote it in a few different ways.

Circle the two true equations. For the equation that is not true, explain to that student why it is not true and how to revise his work.

8. Rewrite the equation $\frac{2}{7} = \frac{6}{21}$ in three other ways to create true equations.

ART SUPPLIES – REVISITED (page 8)

Use a double number line to help you set up proportions and solve problems.

Recall that 3 tubes of artist paint cost \$4.50.

1. How many tubes can you buy for \$12?
 - a. Fill in the boxes above to indicate 12 dollars and x tubes.
 - b. Write a proportion and solve it. Then answer the question.
2. What is the cost of 50 tubes of paint?
3. How many tubes of paint can you buy for \$42?
4. What is the unit price for a tube of paint?

PRACTICE 4 (page 9)

Problems 1-2: Solve using strategies of your choice (tables, unit prices, double number lines, equivalent fractions, proportions). See section 3.5 for different strategies.

1. At the Green Grocer, 2 melons cost \$3.50, and you can purchase as many as you want at this same rate.
 - a. What is the cost per melon?
 - b. How much will 42 melons cost?
 - c. How many melons can be purchased for \$84?
2. At the Grain Grocer, 5 pounds of rice cost \$4.00, and you can purchase any amount at this rate.
 - a. What is the cost per pound?
 - b. How many pounds can be purchased for \$1.00?
 - c. Carlos needs two pounds of rice for a casserole. How much will that cost?

BEST BUY PROBLEMS (page 10)

We will use tables, graphs, and equations to learn more about the behavior of proportional relationships.

1. 0.75 pounds of oranges for \$1.00 or 1.25 pounds of oranges for \$1.00
2. 3 pounds of bananas for \$3.65 or 3 pounds of bananas for \$4.15

Suppose you are running out of your favorite energy snacks, so you compare prices at two stores before making a purchase.

BARTER JACK'S

Healthy Crunch: 2 for \$2.50

Super Bar: 3 for \$3.25

QUIGLEY'S

Healthy Crunch: 2 for \$2.75

Super Bar: 4 for \$3.25

3. Without doing any calculations, explain which store offers the better buy for Healthy Crunch.
4. Without doing any calculations, explain which store offers the better buy for Super Bar.

SOCKS (page 11)

Follow your teacher's directions to explore ways to represent which store has the better buy.

SOX 'R US
CRAZY SOCKS

TORTILLAS (page 12)

FLAT 'N ROUND
3 tortillas for \$0.60

WRAP IT UP
4 tortillas for \$1.00

1. Complete the tables below. Assume each shop will sell any number of tortillas at the rates shown.
2. Label and scale the grid. Graph the data using two different colors.
3. Identify the y -coordinate when $x = 1$.
FLAT 'N ROUND (1,)
WRAP IT UP (1,)
4. Write equations to relate the number of tortillas to the cost.
FLAT 'N ROUND: $y =$ x

WRAP IT UP: $y = \quad x$

5. How are the coordinates for the ordered pairs in problem 3 related to the equations problem 4?
6. How do you know that the point (0, 0) satisfies the equations?
7. Why do these graphs and equations suggest proportional relationships?

PRACTICE 5 (page 13)

A graph for Pizza Palace prices is given. They also offer delivery for any number of pizzas for a fee of \$5.00.

1. Complete the tables.
2. Graph Pizza Palace with delivery.
3. Write equations that relate cost (y) to number of pizzas (x).
Pizza Palace: $y = \quad x$ Pizza Palace (with delivery): $y = \quad x +$
4. Compare unit prices for each store. Use a calculator if needed.

Graph 1. No Delivery / Cost in dollars/ # of pizzas / Unit Price (in dollars/ pizza)

Graph 2. with Delivery / Cost in dollars/ # of pizzas / Unit Price (in dollars/ pizza)

5. Which of these situations represents a proportional relationship? Why?
6. Which of these situations does not represent a proportional relationship? Why?

PRACTICE 6 (page 14)

1. Here are some ticket price options at a local amusement park. Find the unit price for the different plans.
2. Graph the relationship between number of tickets and cost. Be sure to label and scale axes appropriately.
3. Does the ticket pricing represent a proportional relationship? How do you know?
4. Which ticket option would you choose? Why?
5. Here are some costs and quantities for purchasing baseballs. Find the unit price for different quantities.

6. Graph the relationship between number of baseballs and the cost. Be sure to label and scale axes appropriately.
7. Does this represent a proportional relationship? How do you know?

SCALE DRAWINGS (page 15)

We will make and interpret scale drawings. We will learn the meaning of scale factor and scale.

GETTING STARTED

Change all parts of Buddy's face given the following directions to create three more faces. Pay close attention to "width" and "length."

1. Look up scale factor in section 3.5, discuss it in class, and record it in the Word Bank.
2. Whose face represents Buddy's face scaled by a scale factor of 2?
3. Whose face represents Dabney's face scaled by a scale factor of $\frac{1}{2}$?
4. Which two faces look the most alike? and Be prepared to defend your opinion to your classmates.

A BIRD HOUSE (page 16)

This is a birdhouse. Follow your teacher's directions to explore scale. Be sure to record this in your Word bank.

ENLARGEMENTS AND REDUCTIONS (page 17)

Natasha is making scale drawings of a birdhouse she wants to build. She completed scale drawing A on graph paper for the front face of the birdhouse. Then she started drawings B, C, and D.

1. Complete drawings B, C, and D below.
2. Complete the table.

Table Headers: Drawing / Reduction or enlargement compared to drawing A? / Scale factor (multiplier) compared to drawing A / Scale (ratio) compared to drawing A

PRACTICE 7 (page 18)

Another way to describe scale factor is as a percent. For example, a scale factor of 2 could also be described as a scale factor of 200%.

Based on triangle A, complete the table and draw each triangle on the grid paper below.

Chart Header: First Line: Compared to Triangle A

Chart Header: Second Line: Triangle / Scale Factor (as a percent) / Scale factor (as a number) / Scale (ratio) / Enlargement or Reduction / Height (length) "long" / Base (width) "wide"

REVIEW (page 19)

POSTER PROBLEMS: IT'S ABOUT TIME!

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _____.

Each group will have a different colored marker. Our group marker is _____.

Part 2: Do the problems on the posters by following your teacher's directions. Use a calculator as needed.

Chart Header First Line: Poster 1 (or 5) / Poster 2 (or 6) / Poster 3 (or 7) / Poster 4 (or 8)

Chart Header Second Line: A watch gains 2 minutes in 6 hours.) / Mary read 22 pages in 30 minutes./ Betsy cooks 17 hours in a 2-week period. / Hurricane Katrina dropped 14 inches of rain over a 48-hour period.

- A. Copy the fact statement. Find a simple unit rate to describe the situation (This rate may use a different unit of time.)
- B. Assume a proportional relationship. Make a double number line that compares the quantities for different reasonable amounts of time.
- C. Write a question that can be answered using the fact statement.
- D. Answer the question asked in part C.

Part 3: Return to your seats. Work in partners or groups.

1. Look back at each fact statement. Is it reasonable for this relationship to hold up over an extended period of time? Explain.
2. If a watch gains 2 minutes every 6 hours, what time is it?

MATCHING ACTIVITY: NUTS (page 20)

1. Your teacher will give you some cards that represent proportional relationships, except for one error on one card. Work with a partner to match cards with equivalent representations and find the error.
2. What was the error? How do you know? Fix it on the card.
3. Graph the cost vs. quantity for each mixture on the graph using different colors.
4. Do you think the points should be connected? Explain.

SPORTS PLAYING SURFACES (page 21)

You will make scale drawings of sports playing surfaces.

1. Draw a double number line that shows a scale of $\frac{1}{2}$ in. : 10 ft.

Determine the dimensions of each sports surface if the scale is $\frac{1}{2}$ in. : 10 ft. You may want to use the double number line to help you.

Chart Header: Sport Surface / Actual Length / Actual Width / Drawing length / Drawing width

Chart: Sports Surface Column: Soccer Field / Volleyball Court / Football Field / Roller Rink / Bowling Lane / (Your choice – research on internet)

8. Project: Use tools of your choice. Choose two of the sports surfaces above and create scale drawings. You may want to research other features on the internet to include on your scale drawings. Cut them out and label completely.

I made scale drawings for a _____ and a _____.

9. Use your drawings and explain approximately how many copies of your smaller sports surface will fit inside your larger sports surface.

FOCUS ON VOCABULARY (page 22)

Across

- 2 result of a scale factor between 0 and 1
- 4 the graph of a _____ relationship is a straight line through the origin
- 5 _____ lines are parallel lines used to show a proportional relationship (two words)
- 8 result of a scale factor greater than 1
- 9 a multiplier; _____ factor

Down

- 1 cost for 1 (two words)
- 2 comparison of two numbers
- 3 the point (0,0)
- 6 rate for 1; _____ rate
- 7 a strategy for solving proportions; _____ multiplication property

DEFINITIONS, EXPLANATIONS, AND EXAMPLES (page 23)

Header: Word or Phrase / Definition

cross-multiplication property: The cross-multiplication property for proportions states that if $a/b=c/d$, then $ad = bc$. From $2/3=8/12$ we have $3 \times 8 = 2 \times 12$.

proportion: A proportion is an equation stating that the values of two ratios are equal. The equation $3/25=12/100$ is a proportion. It asserts that the values of the ratios $3 : 25$ and $12 : 100$ are equal. The value of both ratios is 0.12.

proportional: Two variables are proportional if the values of one are the same constant multiple of the corresponding values of the other. The variables are said to be in a proportional relationship, and the constant is referred to as the constant of proportionality. If Wrigley eats 3 cups of kibble each day, then the number of cups of kibble is proportional to the number of days. If x is the number of days, and y is the number of cups of kibble, then $y = 3x$. The constant of proportionality is 3.

Scale: In a scale drawing of a figure, the scale is the ratio of lengths in the drawing to lengths in the figure. A blueprint of a house floorplan has a scale of 1 inch to 5 feet, or 1 in : 5 ft. Each inch on the blueprint represents 5 feet in the actual house. A drawing of a lady bug has a scale of 5 cm to 2 millimeters, or 5 cm : 2 mm. Each 5 cm on the drawing represents 2 mm on the actual bug.

DEFINITIONS, EXPLANATIONS, AND EXAMPLES (page 24)

Header: Word or Phrase / Definition

scale drawing: A scale drawing of a geometric figure is a drawing in which all lengths have been multiplied by the same scale factor, while angles remain the same. A blueprint of a house floorplan is a scale drawing.

scale factor: A scale factor is a positive number which multiplies some quantity. To make a scale drawing of a figure, we multiply all lengths by the same scale factor, keeping all angles equal to those in the original figure. If the scale factor is greater than 1, the scale drawing is an enlargement of the actual figure. If the scale factor is between 0 and 1, the scale drawing is a reduction of the actual figure. A drawing of a ladybug has a scale of 5 cm : 2 mm. This is equivalent to 50 mm : 2 mm. The scale factor is $50/2 = 25$. The drawing is an enlargement. A

blueprint of a house floorplan has a scale of 1 in : 5 ft. This is equivalent to 1 in : 60 in. The scale factor is $1/60$. The blueprint is a reduction.

unit price: A unit price is a price for one unit of measure.

unit rate: The unit rate associated with a ratio $a : b$ of two quantities a and b , $b \neq 0$, is the number a/b , to which units may be attached. The ratio of 40 miles each 5 hours has unit rate 8 miles per hour. value of a ratio - The value of the ratio $a : b$ is the number a/b , $b \neq 0$. The value of the ratio $6 : 2$ is $6/2 = 3$. The value of the ratio of 3 to 2 is $3/2 = 1.5$.

Sense-Making Strategies to Solve Proportional Reasoning Problems (page 25)

How much will 5 pencils cost if 8 pencils cost \$4.40?

Strategy 1: Use a “halving” strategy

If 8 pencils cost \$4.40, then 4 pencils cost \$2.20, 2 pencils cost \$1.10, and 1 pencil costs \$0.55.

Therefore, 5 pencils cost $\$0.55 + \$2.20 = \$2.75$.

Strategy 2: Find unit prices

First, find the cost of one pencil. $\$4.40/8 = \0.55 . Then, multiply by 5 to find the cost of 5 pencils, $(\$0.55)(5) = \2.75 .

Sammie can crawl 12 feet in 3 seconds. At this rate, how far can she crawl in 1 1/2 minutes?

Strategy 1: Make a table

Sammie can crawl 360 feet in 1 1/2 minutes.

Strategy 2: Make a Double Number Line

1 1/2 minutes = 90 seconds. Sammie can crawl 360 feet in 1 1/2 minutes.

Setting Up Proportions (page 26)

Here are some ways to set up a proportion to solve a problem.

If 2 pencils cost \$0.64, how much will 5 pencils cost?

Strategy 1: Compare rates. This is sometimes referred to as a “between” proportion because the ratios contain different units (i.e., between two units). Between proportions can be read directly from a double number line.

Organize known information on a double number line if desired. From the diagram we see:

Then, equate the two expressions and solve for x . $x = 1.60$ dollars for 5 pencils.

Note: The equation $5/x=2/0.64$ is another valid “between” proportion for this problem.

Strategy 2: Compare like units. This is sometimes referred to as a “within” proportion because the ratios contain the same units (i.e., within the same unit).

Use a double number line to organize information if you wish.

Create one ratio based on corresponding costs and another ratio based on the corresponding numbers of pencils.

Then, equate the two ratios, and solve for x . $x = 1.60$ dollars for 5 pencils.

Note: The equation $x/0.64=5/2$ is another valid “within” proportion for this problem.

Some Properties Relevant to Solving Proportions (page 27)

Here are some important properties of arithmetic and equality related to proportions.

- The multiplication property of equality states that equals multiplied by equals are equal. Thus, if $a = b$ and $c = d$, then $ac = bd$.

Example: If $6/2 = 3$ and $5 = 9 - 4$, then $6/2 (5) = 3(9 - 4)$.

The fraction-inverse property for proportions states that if two nonzero fractions are equal, then their inverses are equal. That is, if ...

- The cross-multiplication property for proportions states that if $a/b=c/d$, then $ad = bc$ ($b \neq 0$, $d \neq 0$).

This can be remembered with the diagram:...

Example: If $5/7=12/x$, then $5 \bullet x = 7 \bullet 12$. To see that this property is reasonable, try simple numbers: If $3/4=6/8$, then $3 \bullet 8 = 4 \bullet 6$.

Applying Properties to Solve Proportions

Strategy 1:
Multiplication Property of Equality

Solve for x :

Strategy 2:

Cross-Multiplication Property

Solve for x :

Testing for a Proportional Relationship (page 28)

Here are three ways to test if two variables are in a proportional relationship:

- The values of the ratios (unit rates or unit prices) created by data pairs are the same.
- An equation in the form $y = kx$ fits all corresponding data pairs.

Graphed data pairs fall on a line through the origin $(0, 0)$.

Alexa buys tickets when she goes to the amusement park. This chart shows the costs for different quantities of tickets.

Chart First Column: # of tickets / total cost / cost per ticket

Since the costs per ticket (unit prices) are not the same, ticket purchasing at this amusement park does not represent a proportional relationship.

Antonio kept track of the number of miles he traveled each time he filled his tank with gas. Here is some data.

Chart First Column: number of miles / number of gallons / miles per gallon

Since the miles per gallon (unit rates) created by the data pairs is the same, this situation represents quantities in a proportional relationship.

Furthermore,

Let x = the number of gallons

Let y = the number of miles

The data fits the equation $y = 25x$ (an equation in the form $y = kx$), which is an equation that represents a proportional relationship.

Finally, if the points for (gallons, miles) are graphed, they will fall on a line through the origin $(0,0)$.

Multiple Representations and Proportional Relationships

(Page 29)

Suppose 4 balloons cost \$6.00 and each balloon is the same price. Here are some strategies for representing this proportional relationship.

Strategy 1: Tables

Create a table to calculate unit rates. If the unit rates are the same, the variables are in a proportional relationship.

Strategy 2: Graphs

A straight line through the origin indicates quantities in a proportional relationship.

Strategy 3: Equations

An equation of the form $y = kx$ indicates quantities in a proportional relationship. In this case,

y = cost in dollars

x = number of balloons

k = cost per balloon (unit price)

To determine the unit price, create a ratio whose value is: 6 dollars/ 4balloons = 1.50 dollars/balloons.

Therefore, $k = 1.50$ dollars per balloon, and $y = 1.50x$.

This equation expresses the output as a constant multiple of the input, showing that the relationship is proportional.

Scale Drawings (page 30)

A scale drawing of a geometric figure is a drawing in which all distances have been multiplied by the same scale factor, while angles remain the same. If the scale factor is greater than 1, the figure is expanded (enlarged), and if the scale factor is between 0 and 1, the figure is reduced in size.

The ratio of lengths in the drawing to lengths in the actual figure is the scale of the drawing. A scale of 1:1 implies that the drawing is the same as the actual object. A scale 1 : 2 implies that the drawing is smaller (half the size) than the actual object (in other words, the dimensions are multiplied by a scale factor of 0.5).

- To make Triangle B below, multiply each dimension of Triangle A by a scale factor of 3. Triangle B is a 300% enlargement of Triangle A. The scale is 1 : 3.

- To make Triangle C below, multiply each dimension of Triangle A by a scale factor of $\frac{1}{2}$. Triangle C is a 50% reduction of triangle A. The scale is 2 : 1.

Scale Drawing of a Flag

A scale drawing of a geometric figure is a drawing in which all distances have been multiplied by the same scale factor while angles remain the same. If the scale factor is greater than 1, the figure is expanded (enlarged), and if the scale factor is between 0 and 1, the figure is reduced in size.

The flag of Mexico is composed of three stripes (green, white, and red) that divide the flag into thirds. The national coat of arms is in the center of the white stripe. Pictured below is a (gray) scale drawing of the flag.

Suppose the original flag is 36 inches by 24 inches, and the scale drawing is 1.5 inches by 1 inch.

This scale may be represented as a ratio:

scale drawing : actual flag
1.5 inch : 36 inches
1 inch : 24 inches
1 : 24

The scale drawing is a reduction of the flag, with scale factor (value of the ratio) $\frac{1}{24}$.

The ratio “1 inch on the scale drawing represents 24 inches on the actual flag” is sometimes written with the technically incorrect, but convenient notation “1 inch = 24 inches.” We will not do this because everyone knows that 1 inch does not really equal 24 inches!